1 Introduction to Electromagnetic Waves

**electromagnetic wave**: time-varying electric and magnetic field propagating through space from one region to another even when there is no matter in the intervening region (versus water waves, sound waves)

*Maxwell’s Equations (one form):*

\[
\begin{align*}
\nabla \cdot \vec{E} &= \frac{\rho}{\epsilon_0} \\
\nabla \cdot \vec{B} &= 0 \\
\nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\
\nabla \times \vec{B} &= \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}
\end{align*}
\]

2 Speed of an Electromagnetic Wave

**wave front**: the boundary of a wave

E = cB, only in SI units

\[
c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}
\]

*Characteristics of electromagnetic waves in vacuum:*

- the wave is transverse; both \( \vec{E} \) and \( \vec{B} \) are perpendicular to the direction of propagation of the wave and to each other
- there is a definite ratio between the magnitudes of \( \vec{E} \) and \( \vec{B} \); \( E = cB \)
- the wave travels in vacuum with a definite and unchanging speed \( c \)
- electromagnetic waves require no medium

3 The Electromagnetic Spectrum

**electromagnetic spectrum**: the broad spectrum of wavelengths covered by electromagnetic waves including radio, TV, and cellular phone transmission, microwaves, visible light, infrared and ultraviolet light, radiation, x rays and gamma rays.

*speed of electromagnetic waves in vacuum:*

\[
c = \lambda f
\]

where \( \lambda \) is the wavelength and \( f \) is the frequency of the wave

*monochromatic*: single-color light
4 Sinusoidal Waves

*plane wave*: at any instant the fields are uniform over any plane perpendicular to the direction of propagation. One form of the wave function for a transverse wave traveling to the right along a stretched string:

$$y(x, t) = A \sin (\omega t - kx)$$

where $y$ is the transverse displacement from its equilibrium position at time $t$ of a point with coordinate $x$ on the string where
- $A$ is the maximum displacement or amplitude of the wave
- $\omega$ is the angular frequency ($2\pi$ times the frequency $f$)
- $k$ is the wave number or propagation constant ($2\pi/\lambda$) where
- $\lambda$ is the wavelength

Where $E$ and $B$ are the instantaneous values of the electric and magnetic fields respectively, and $E_{\text{max}}$ and $B_{\text{max}}$ are the amplitudes of those fields

$$E = E_{\text{max}} \sin(\omega t - kx)$$
$$B = B_{\text{max}} \sin(\omega t - kx)$$

thus the sinusoidal oscillations of $\vec{E}$ and $\vec{B}$ are in phase and the amplitudes are related by

$$E_{\text{max}} = cB_{\text{max}}$$

*linearly polarized*: a wave whose $\vec{E}$ field always lies along the same line.

5 Energy in Electromagnetic Waves

*energy density*: energy per unit volume.

**Energy density in electric and magnetic fields:**

The energy density $u$ (energy per unit volume) in a region of empty space where electric and magnetic fields are present is

$$u = \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2} \mu_0 B^2.$$  

The two field magnitudes are related by

$$B = \frac{E}{c} = \sqrt{\varepsilon_0 \mu_0} E.$$  

Thus the energy density $u$ can also be expressed as

$$u = \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2} \mu_0 (\sqrt{\varepsilon_0 \mu_0} E)^2 = \varepsilon_0 E^2$$

and the energy density associated with the $\vec{E}$ field is equal to the that of the $\vec{B}$.  

**Intensity**: The average power per unit area in an electromagnetic wave; \( I = S_{\text{average}} \).

\[
I = S_{\text{av}} = \frac{1}{2} \sqrt{\frac{\epsilon_0}{\mu_0}} E_{\text{max}}^2 = \frac{1}{2} \frac{\epsilon_0 c}{\mu_0} E_{\text{max}}^2 = \frac{E_{\text{max}} B_{\text{max}}}{2\mu_0}.
\]

**Radiation pressure**: The force due to the absorption of an electromagnetic wave on a surface perpendicular to the direction of the propagation of the wave; the rate of change of the momentum \( p \) per unit area or \( I/c \) where \( c \) is the speed of light.

6 **Nature of Light**

“What’s in a name? that which we call a rose
By any other name would smell as sweet;” Sir William Shakespeare

but can a rose be something else entirely?

**Light—both wave and particle**

- Wave (continuous) best for understanding propagation (path)
- Photons or quanta (discrete bundles of energy) best for understanding absorption by atoms and nuclei (energy)
- Quantum electrodynamics: A comprehensive theory that includes both wave and particles properties

**Fundamental sources of all electromagnetic radiation (EMR)** are electric charges in accelerated motion

**Wave front**: The locus of all adjacent points at which the phase of vibration of the wave is the same.

**Ray**: An imaginary line along the direction of travel of the wave.

**Geometric optics**: Branch of optics using the ray behavior of light

**Physical optics**: Branch of optics using wave behavior
7 Reflection and Refraction

reflection: light that scattered off of the incident material.
refraction: light that is transmitted through the incident material.
specular reflection: reflection at a definite angle from a very smooth surface.
diffuse reflection: scattered reflection from a rough surface.

index of refraction

The index of refraction of an optical material, denoted as \( n \), is the ratio of the speed of light in vacuum (\( c \)) to the speed of light in the material (\( v \)):

\[
  n = \frac{c}{v}
\]

Light always travels more slowly in a material than in vacuum, so \( n \) for any material is always greater than one. For vacuum, \( n=1 \) by definition.

Principles of geometric optics

1. The incident, reflected, and refracted rays, and the normal to the surface, all lie in the same plane.
2. The angle of reflection \( \Theta_r \) is equal to the angle of incidence \( \Theta_a \) for all wavelengths and for any pair of substances; \( \Theta_r = \Theta_a \). Law of reflection.
3. With qualifiers, the ratio of the sines of the angles \( \Theta_a \) and \( \Theta_b \), where both angles are measured from the normal to the surface, is equal to the inverse ratio of the two indexes of refraction:

\[
  \frac{\sin \Theta_a}{\sin \Theta_b} = \frac{n_b}{n_a} \quad \text{or} \quad n_a \sin \Theta_a = n_b \sin \Theta_b.
\]

Law of refraction or Snell's law.

The wavelength \( \lambda \) of the light in a material is less than its wavelength \( \lambda_0 \) in a vacuum by the factor \( n \):

\[
  \lambda = \frac{\lambda_0}{n}.
\]

8 Total Internal Reflection

critical angle: the angle of incidence from which the refracted ray emerges tangent to the surface.

Total internal reflection

When a ray traveling in a material \( a \) with index of refraction \( n_a \) reaches an interface with a material \( b \) having index \( n_b \), where \( n_b < n_a \), it is totally reflected back into material \( a \) if the angle incidence is greater than the critical angle given by

\[
  \sin \Theta_{\text{critical}} = \frac{n_b}{n_a}
\]

9 Dispersion

dispersion: the dependence of wave speed and index of refraction on wavelength
10 Polarization

*linearly polarized*: a wave with displacement in only one plane

*polarizing filter or polarizer*: a filter that polarizes a wave in a certain direction

*dichroism*: the selective absorption of one of the polarized components much more strongly than the other

*polarizing axis*: the axis in which the light is polarized parallel to

---

**Light transmitted by polarizing filter**

When linearly polarized light strikes a polarizing filter with its axis at an angle $\phi$ to the direction of polarization, the intensity of the transmitted light is

$$I = I_{\text{max}} \cos^2 \phi,$$

where $I_{\text{max}}$ is the maximum intensity of the light transmitted (at $\phi = 0$) and $I$ is the amount transmitted at angle $\phi$. This is known as Malus’s Law.

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*plane of incidence*: the plane containing the incident and reflected rays and the normal to the surface.

*polarizing angle*: the angle for which $\vec{E}$ is perpendicular to the plane of incidence (and parallel to the reflecting surface), thus the reflected light is linearly polarized perpendicular to the plane of incidence.

*Brewster’s law*:

$$\frac{\sin \Theta_p}{\cos \Theta_p} = \tan \Theta_p = \frac{n_b}{n_a}$$

*photoelasticity*: the property of certain materials, when placed under mechanical stress, to have their index of refraction different for different planes of polarization.

11 Huygens Principle

**Huygen’s Principle**

Every point of a wave front may be considered the source of secondary wavelets that spread out in all directions with a speed equal to the speed of propagation of the wave.

12 Scattering of Light

*scattering*:


13 Links

Poster of EM spectrum pdf right click and download; extensive Berkeley EM scale pdf
Mathematica Demonstrations: Electromagnetic Wave

EM radiation in the news:
Supermassive Black holes and from Muse Supermassive Black hole
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15 Constants

<table>
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<tr>
<th>Constant</th>
<th>Value</th>
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<tr>
<td>speed of light in a vacuum</td>
<td>$c = 2.99792458 \times 10^8$ m s$^{-1}$</td>
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<tr>
<td>mass of the electron</td>
<td>$m_e = 9.1093826(16) \times 10^{-31}$ kg</td>
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<tr>
<td>mass of the proton</td>
<td>$m_p = 1.67262171(29) \times 10^{-27}$ kg</td>
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<tr>
<td>mass of the neutron</td>
<td>$m_n = 1.67492728(29) \times 10^{-27}$ kg</td>
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<tr>
<td>electric force constant or</td>
<td>$k = 8.987551789 \times 10^9$ N m$^2$/C$^2$</td>
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<td>electrostatic constant</td>
<td>$k = \frac{1}{4\pi\epsilon_0}$</td>
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<td>vacuum permittivity</td>
<td>$\epsilon_0 = 8.854 \times 10^{-12}$ C$^2$/N m$^2$</td>
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<td>fundamental unit of charge</td>
<td>$e = 1.602176565(35) \times 10^{-19}$ C</td>
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<td>vacuum permeability</td>
<td>$\mu_0 = 4\pi \times 10^{-7}$ N/A$^2$</td>
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